Toward Data-Driven Models of Legged Locomotion using Harmonic Transfer Functions

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Abstract*—There are limitations on the extent to which manually constructed mathematical models can capture relevant aspects of legged locomotion. Even simple models for basic behaviours such as running involve non-integrable dynamics, requiring the use of possibly inaccurate approximations in the design of model-based controllers. In this study, we show how data-driven frequency domain system identification methods can be used to obtain input–output characteristics for a class of dynamical systems around their limit cycles, with hybrid structural properties similar to those observed in legged locomotion systems. Under certain assumptions, we can approximate hybrid dynamics of such systems around their limit cycle as a piecewise smooth linear time periodic system (LTP), further approximated as a time-periodic, piecewise LTI system to reduce parametric degrees of freedom in the identification process. In this paper, we use a simple one-dimensional hybrid model in which a limit-cycle is induced through the actions of a linear actuator to illustrate the details of our method. We first derive theoretical harmonic transfer functions (HTFs) of our example model. We then excite the model with small chirp signals to introduce perturbations around its limit-cycle and present systematic identification results to estimate the HTFs for this model. Comparison between the data-driven HTFs model and its theoretical prediction illustrates the potential effectiveness of such empirical identification methods in legged locomotion.

I. INTRODUCTION

Legged locomotion emerges from a staggering diversity of animal and robot morphologies and gaits, and modeling locomotor dynamics remains a grand challenge in both biology and robotics [1, 2]. Running behaviors, in particular, are commonly represented by relatively simple spring–mass models such as the Spring-Loaded Inverted Pendulum (SLIP) model [3]. A common feature of such models, however, is that their hybrid system dynamics involve intermittent foot contact with the ground, alternating between flight and stance phases during locomotion. Despite the presence of seemingly simple models for basic behaviors such as running and walking, the hybrid dynamics associated with these behaviors can be rather complex, with non-integrable parts such as the stance phase [4]. Given the utility of having accurate models and associated analytic solutions in constructing high performance controllers for nonlinear systems, substantial effort has been devoted to the construction of approximate solutions to such non-integrable hybrid models [5–8].

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When accurate analytical solutions to the dynamics of a legged platform are available [7], their structure can be exploited to yield effective solutions for system identification and adaptive control [9]. Despite our previous studies showing how accurate such models may be, there will always be unmodeled components in the physical system, resulting in discrepancies between the model and experiments [10]. Attempts to manually incorporate these effects into the model is daunting at best, and often impossible. Consequently, we propose an alternative method in this study, namely using data-driven system identification methods to derive an input–output transfer function for such hybrid legged locomotion behaviors, thereby eliminating the need to manually construct an explicit mathematical model for the system.

Our main goal in this study is to provide a system identification framework applicable to a useful (although not comprehensive) class of legged locomotion models [7], and possibly more complex robotic systems [11]. Our approach is based on considering legged locomotion as a hybrid nonlinear dynamical system with a stable periodic orbit (limit-cycle), corresponding to the locomotor behavior of interest. We introduce a formulation that addresses the input–output system identification problem in the frequency domain for a sub-class of hybrid legged locomotion models. More specifically, following certain assumptions on the hybrid dynamics of legged systems, we approximate their hybrid dynamics around the limit-cycle as a linear time-periodic system (LTP). However, this first LTP approximation is infinite dimensional, making parametric identification challenging. We hence further approximate the dynamics as a finite dimensional *piecewise LTI* system (maintaining its LTP nature), thereby limiting the parametric degrees of freedom while enabling a practical identification framework.

Existing studies on system identification of LTP systems focus on modeling these systems as multi-input single-output LTI systems. This approach is based on the concept of harmonic transfer functions (HTFs) [12], which are infinite-dimensional operators that are analogous to frequency response functions for LTI systems. An identification strategy for such systems was developed in [13] using power spectral density and cross spectral density functions. A similar method was used in [14] considering the effects of noise in both input and output measurements. Different than these studies, local polynomial methods and lifting approaches were also used for the identification of HTFs for multi-input single-output models of LTP systems [15]. Ankarali and Cowan [16] developed a similar system identification method for hybrid systems with periodic orbits using discrete-time HTFs.

Our contributions in this paper focus on representing the dynamics of legged locomotion as a linear time periodic system, thereby enabling the use of the system identification method proposed in [13] for such systems. We achieve this by using a new phase definition in identifying the HTFs, illustrated in the context of a simplified model designed to mirror structural properties of legged locomotion models. When the problem is approached as a grey-box model with finite parameters (piecewise LTI), it suffices to non-parametrically estimate a finite number of harmonics, to which we later fit parametric models.

II. Background: Harmonic Transfer Functions

Many linear time periodic (LTP) systems are represented in the form

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) + D(t)u(t), \end{aligned} \tag{1}$$

where all system matrices are periodic with a common period T. This time dependence makes system identification more challenging than it is for LTI systems. The key reason behind this difficulty is that LTP systems produce output spectra that include multiple (possibly infinite) harmonics of the input stimulus, each with possibly different magnitude and phase in steady state.

Motivated by this problem, Wereley [17] proposed a linear one-to-one mapping between the coefficients of an exponentially modulated periodic (EMP) signal at the input of an LTP system to the coefficients of an EMP signal at its output. This linear operator, that maps the input harmonics to the output harmonics of an LTP system, is called a Harmonic Transfer Function (HTF) [12].

One way of deriving HTFs begins with obtaining the individual Fourier series expansions of the T-periodic (pumping frequency $\omega_p = 2\pi/T$) system matrices in (1). Based on these expansions, one can obtain the harmonic state space ($\mathcal{HSS}$) model of the LTP system by using the principle of harmonic balance:

$$\begin{aligned} s\mathcal{X} &= (\mathcal{A} - \mathcal{N})\mathcal{X} + \mathcal{B}\mathcal{U} \\ \mathcal{Y} &= \mathcal{C}\mathcal{X} + \mathcal{D}\mathcal{U}, \end{aligned} \tag{2}$$

where $\mathcal{HSS}$ parameters are doubly infinite Toeplitz structures corresponding to initial system matrices in (1). The details about transforming the classical state space representation to an $\mathcal{HSS}$ model can be found in [12]. Note that a new matrix, $\mathcal{N} := \text{blockdiag}\{jn\omega_p I\}, \forall n \in Z$, appears in the $\mathcal{HSS}$ model to modulate the input frequency to different harmonic frequencies.

The $\mathcal{HSS}$ model in (2) is useful to determine an explicit input–output functional relationship between the input and output signals. This relationship is represented by the harmonic transfer functions, $G(s)$, which can be computed as

$$G = \mathcal{C}[s\mathcal{I} - (\mathcal{A} - \mathcal{N})]^{-1}\mathcal{B} + \mathcal{D} \ , \tag{3}$$

as long as the inverse within this equation exists. The resulting $G(s)$ is a doubly infinite Toeplitz matrix, whose (k, l) entry has the form $G_{k-l}(s + kj\omega_p)$, where $k, l \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$; for more details see [12]. Note that the derivation of HTFs using this approach requires a priori knowledge of system dynamics. Our goal in this work is to estimate HTFs using input–output data. Will will use the above formulation for the parametric identification phase, which is the second phase of our data-driven system identification method (see Section IV-C).

The data-driven LTP system identification method that we adopt in this paper requires the truncation of the number of HTFs to be estimated to support the implementation on computers. To illustrate, suppose that our truncation consists of three different harmonic transfer functions, $\hat{G}_0$, $\hat{G}_{-1}$ and $\hat{G}_1$, each corresponding to a different frequency component of the output. The output can then be expressed as

$$\begin{aligned} \hat{Y}(j\omega) \quad = \quad & \hat{G}_0(j\omega)U(j\omega) + \hat{G}_{-1}(j\omega)U(j\omega + j\omega_p) \\ & + \hat{G}_1(j\omega)U(j\omega - j\omega_p). \end{aligned} \tag{4}$$

In this new formulation, identification problem is reduced to estimating a finite number of HTFs for each specific frequency by using the available input–output data at this specific frequency.

Note that the choice of input signal significantly affects the system identification performance for both LTI and LTP systems. However this choice is more nuanced for LTP systems, because of the specific structure of HTFs [14, 16]. Siddiqi [13] uses a single input sequence signal for system identification, constructed by concatenating phase shifted copies of a single waveform on the input evenly separated by delays within the system period. A complete characterization of system dynamics is possible with this method since different modes of the system were activated through the use of phase-shifted copies of a single waveform.

The second issue is the need to excite all frequency components within the system by providing input signals with a sufficiently wide frequency spectrum. This can be accomplished through the use of chirp signals, whose frequency varies with time. The use of chirp input signals, combined with the idea of supplying multiple, phase-shifted input sequences allows us to obtain sufficiently rich input–output data to support the system identification process.

Using this input–output data, one can estimate the HTFs of the system, so that the error between the actual and estimated output is minimized. Afreen [13] also adds a cost (penalty) for the curvatures in the estimated HTFs in order to generate smooth non-parametric transfer function models. Then the HTFs can be computed as

$$\mathbf{\hat{G}} = (\mathbf{U^T U} + \alpha \mathbf{D^4})^{-1} \mathbf{U^T Y} \ , \tag{5}$$

where α and $\mathbf{D^2}$ are the weight of the curvature penalty and the second difference operator respectively.

III. Representation of Legged Locomotion as a Hybrid Dynamical System

Our goal in this study is to provide a system identification framework for a class of models related to legged locomotion using harmonic transfer functions (HTFs). For the present paper, we limit ourselves to "clock-driven" locomotion models, representative of controllers used by a wide variety of existing robots [11, 18], with open-loop central pattern generators (CPG) coordinating control actions to achieve time periodic behaviour. This will allow us to directly use time periodicity in our LTP analysis, while eliminating a variety of complications associated with estimating the phase [19].

A. Modeling Framework for Hybrid Systems

Legged systems are often modeled using hybrid dynamics due to intermittent foot contact with the ground, which cannot be represented with a single, smooth dynamical flow. In the broadest sense, a hybrid dynamical system is a set of smooth flows together with discrete transitions (and associated transformations) between these flows triggered by intersections of system trajectories with sub-manifolds of the continuous state space [20]. These flows are called *charts*, indexed with unique labels $\mathcal{I} := \{0, \cdots, d\}$ each with possibly different equations of motion. Along its trajectories, a hybrid system transitions from one chart to another, with each transition defined by the zero crossing of a *threshold function*. For each source chart $\alpha \in \mathcal{I}$ and destination chart $\beta \in \mathcal{I}$, the threshold function h_α^β defines the transition from chart α to chart β. An example transition graph for a hybrid dynamical system is illustrated in Fig. 1.

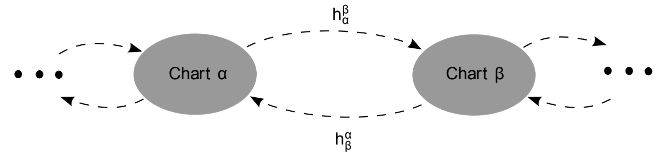

Fig. 1. A simple state transition graph for a hybrid dynamical system.

Since we are interested in the local behaviour around the limit-cycle, we assume that there is only one transition function associated with each chart.[1] We further assume that system trajectories are continuous at transitions, meaning that system states do not experience discrete changes coincident with chart transitions. As a final note, we assume that the hybrid dynamical system we consider has an isolated periodic orbit ensuring that chart transitions within the limit cycle are also periodic and consistent.

It is important to note that these assumptions are generally satisfied by models of common locomotory behaviors such as running and walking [7, 21] as well as a wide range of legged robots for which leg masses are negligible compared to the dynamics of a larger body [11, 18]. Consequently, the system identification methods we introduce will remain applicable to systems other than the simplified example we will present in this paper.

B. Modeling Legged Locomotion as a Linear Time Periodic System

For clarity, we limit our focus in this section to an example hybrid dynamical system with only two charts, $\mathcal{I} = \{0, 1\}$, designed to capture stance and flight phases of simple spring-mass models of locomotion. Based on a clock driven assumption, for each $i \in \mathcal{I}$ the continuous dynamics can be represented with

$$\begin{aligned} \dot{\phi} &= 1 \\ \dot{q}_i &= f_i(q, \phi, u) \ , \\ q_i &\in \mathbb{R}^n \end{aligned} \tag{6}$$

[1]This approach does not apply to gaits such as pronking that nominally involve multiple legs making contact with the ground at the same time when on the limit cycle, because small deviations from the limit cycle can lead to different touch-down order between legs, violating our assumption.

and let the associated threshold function be $h_i^{\mathrm{mod}(i+1,2)}(q)$. The transition map associated with each hybrid event is simply the identity map, $q_i \mapsto q_i$, due to the continuity assumption. Our linearization of these hybrid dynamics towards an LTP approximation assumes that these transition times, $\hat{t}$, zero crossings of $h_0^1(q)$ and $h_1^0(q)$, maintain their periodicity and offsets within the period in close proximity of the limit-cycle, resulting in the following form of the nonlinear dynamics

$$\dot{\phi} = 1 \tag{7}$$

$$\dot{q} \approx \begin{cases} f_0(q, \phi, u) \ , \ \text{if } \mathrm{mod}(t, T) \in [0, \hat{t}) \\ f_1(q, \phi, u) \ , \ \text{if } \mathrm{mod}(t, T) \in [\hat{t}, T) \end{cases} . \tag{8}$$

Assuming that the system given above has a limit cycle $\bar{q}(t)$ with a period T, linearization around $\bar{q}(t)$ yields the piecewise smooth LTP system

$$\dot{x}(t) = \begin{cases} A_0(t)x(t) + B_0(t)u(t), \ \text{if } \mathrm{mod}(t, T) \in [0, \hat{t}) \\ A_1(t)x(t) + B_1(t)u(t), \ \text{if } \mathrm{mod}(t, T) \in [\hat{t}, T) \end{cases}$$

where

$$A_0(t) := \left[\frac{\partial f_0}{\partial q}\right]_{\substack{q(t) \, = \, \bar{q}(t) \\ u(t) \, = \, 0}}, B_0(t) := \left[\frac{\partial f_0}{\partial u}\right]_{\substack{q(t) \, = \, \bar{q}(t) \\ u(t) \, = \, 0}},$$

$$A_1(t) := \left[\frac{\partial f_1}{\partial q}\right]_{\substack{q(t) \, = \, \bar{q}(t) \\ u(t) \, = \, 0}}, B_1(t) := \left[\frac{\partial f_1}{\partial u}\right]_{\substack{q(t) \, = \, \bar{q}(t) \\ u(t) \, = \, 0}}.$$

It is natural to assume that direct measurement of all $x(t)$ may not be available or we may only measure a subset of $x(t)$. Consequently, we also define a time-periodic output equation as in the form (10).

Since system matrices $A_i(t)$, $B_i(t)$, $C_i(t)$ and $D_i(t)$ with $i \in \{0, 1\}$ are time parametrized functions, the system has infinite parametric degrees of freedom, making parametric system identification challenging even when HTFs are used. At this point, we hypothesize that for hybrid systems, the variability within a chart is small compared to the change between charts and we approximate the LTP dynamics using a piecewise LTI approximation that preserves the LTP structure of the system. The LTP equations of motion then take the form

$$\dot{x}(t) \approx \begin{cases} A_0 x(t) + B_0 u(t), \ \text{if } \mathrm{mod}(t, T) \in [0, \hat{t}) \\ A_1 x(t) + B_1 u(t), \ \text{if } \mathrm{mod}(t, T) \in [\hat{t}, T) \end{cases} \tag{9}$$

$$y(t) \approx \begin{cases} C_0 x(t) + D_0 u(t), \ \text{if } \mathrm{mod}(t, T) \in [0, \hat{t}) \\ C_1 x(t) + D_1 u(t), \ \text{if } \mathrm{mod}(t, T) \in [\hat{t}, T) \end{cases} \tag{10}$$

The formulation above constitutes the basis of our framework for analyzing and identifying clock-driven legged locomotion models.

IV. Simplified Legged Locomotion Model with Hybrid System Dynamics

In this section, we describe a simple, vertically constrained spring-mass-damper system that possesses hybrid structural properties similar to the extensively studied Spring-Loaded Inverted Pendulum (SLIP) model for running behaviors. This will provide a simple example to illustrate the application of our system identification method to such systems.

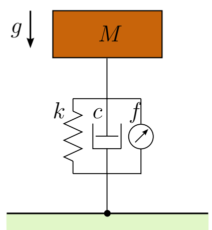

Fig. 2. Simplified leg model.

A. System Dynamics

Fig. 2 illustrates the vertical leg model we focus on in this section. It consists of a mass attached to a leg with a spring-damper mechanism as well as a force transducer. Unlike the SLIP model, we assume that the toe is permanently affixed on the ground. Nevertheless, we recover the hybrid nature of locomotory gaits by assuming that the damper is turned on during a "stance phase" (lossy) and off during a "flight phase" (lossless). This construction recovers the hybrid nature of the dynamics, while allowing active input throughout the entire trajectory to support the generation of system identification data, as well as admitting theoretical computation of its HTFs for a comparative investigation.

We use the force transducer f in this system for two purposes. Firstly, active energy input to the system must be provided to maintain the limit cycle and compensate for energy losses due to the presence of damping. Second, it will be used as an exogenous input to the system to support the system identification process. Many physical legged platforms include similar active components in their legs to regulate their mechanical energy [22]. Notwithstanding differences in how these actuators are incorporated into the system, they can all be used as the necessary exogenous inputs to perform system identification. A similar model was also investigated in [15] but using an additional nonlinear spring for energy regulation.

The equations of motion for this simplified legged locomotion model are given by

$$m\ddot{x} = \begin{cases} -mg - c\dot{x} - k(x - x_0) + f(t), & \text{if } \dot{x} > 0 \\ -mg - k(x - x_0) + f(t), & \text{otherwise.} \end{cases} \tag{11}$$

The lossy and lossless dynamics in (11) correspond to different charts in Fig. 1 and zero crossings of $\dot{x}$ represent threshold functions for both phases.

Our illustrative examples use the parameters $g = 9.81$, $k = 200$, $c = 2$, $m = 1$ and $x_0 = 0.2$, chosen to be similar to the parameters of a vertical hopper platform in our laboratory [23]. As noted above, we choose the linear actuator input $f(t) = f_0(t) + u(t)$, consisting of a forcing term $f_0(t)$ to compensate for energy losses, and a chirp signal $u(t)$ to introduce small periodic perturbations for system identification.

B. Theoretical Computation of Harmonic Transfer Functions

In this section, we derive the theoretical approximations of the HTFs for our model as outlined in Section II for validation purposes and also for use in the parametric identification phase. To accomplish this goal, we first assume that the forcing input $f_0(t)$ is appropriately chosen to induce an asymptotically stable limit cycle for this system. For example, our simple leg model achieves a stable limit cycle with $f_0(t) = \cos(2\pi t)$. At this point, changing into error coordinates away from the limit cycle with $\xi = x(t) - \bar{x}(t)$, and substituting into (11), the equations of motion take the form

$$\ddot{\xi} = \begin{cases} -c\dot{\xi} - k\xi, & \text{if } \dot{\xi} + \dot{\bar{x}}(t) > 0 \\ -k\xi, & \text{otherwise} \end{cases} \tag{12}$$

Due to the simplicity of the dynamics, this corresponds to a piecewise LTI system without necessitating any additional approximations, taking the form

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k & -cs(\dot{\xi}, t) \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \tag{13}$$

where the hybrid nature of the system is captured by the flag $s(\dot{\xi}, t)$, with $s = 1$, when $\dot{\xi} + \dot{\bar{x}}(t) > 0$ and $s = 0$ otherwise.

We now need to represent this piecewise LTI system as a linear time periodic system. However, even though the binary valued function $s(\dot{\xi}, t)$ can be considered time-periodic on the limit cycle itself, this is not the case for trajectories away from the limit cycle. To proceed, we hence assume that input induced perturbations are small, and that the binary valued function $s(\dot{\xi}, t)$ maintains its period and becomes strictly time dependent rather than state dependent, taking the form $s(\dot{\xi}, t) \approx s(t)$. We now can perform a Fourier series expansion on $s(t)$ by treating it as a square wave with an offset to obtain a linear time periodic system in the form

$$\begin{aligned} \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -k & -cs(t) \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} . \end{aligned} \tag{14}$$

Plugging these equations into the HTF framework described in [17], yields analytic solutions to the harmonic transfer functions. We omit the details of this derivation due to space considerations, but use the resulting analytic solutions for the HTFs up to $n_h = 10$ to evaluate the output of our system identification method.

C. Data-Driven Identification of Harmonic Transfer Functions

In this section, we obtain harmonic transfer functions corresponding to the linearized dynamics of (14) by using input–output data without assuming prior knowledge of the state space model. Using $f_0(t) = \cos(2\pi t)$ and $u(t) = 0$ for 30 cycles without a perturbation, our example system stabilized to a limit cycle $\bar{x}(t)$ with a period $T = 1s$. We use the 30^{th} period as the numerical limit cycle of the nonlinear system and subtract it from the trajectories of subsequent experiments to obtain the error function ξ_1.

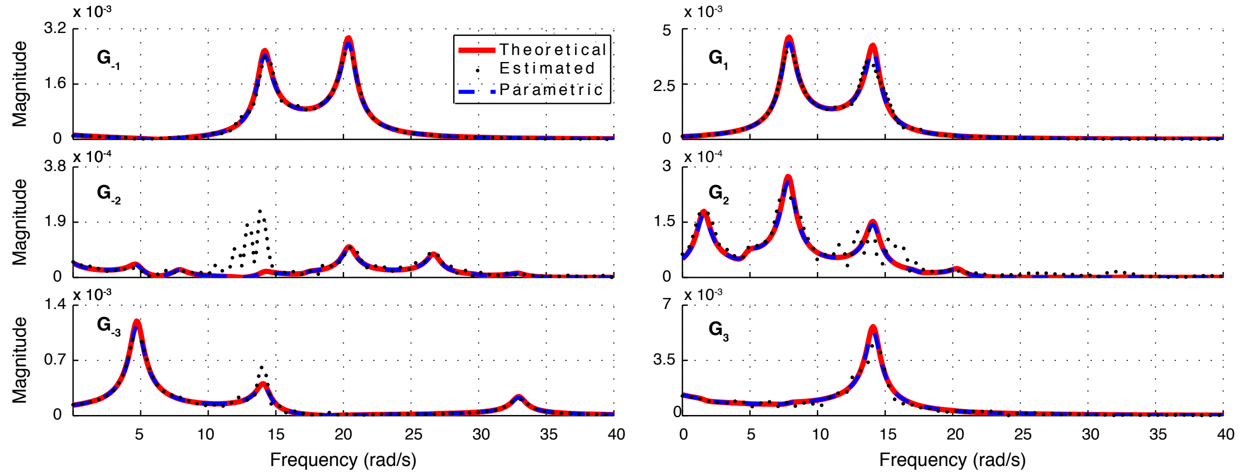

Fig. 3. Estimation results for the higher order harmonics.

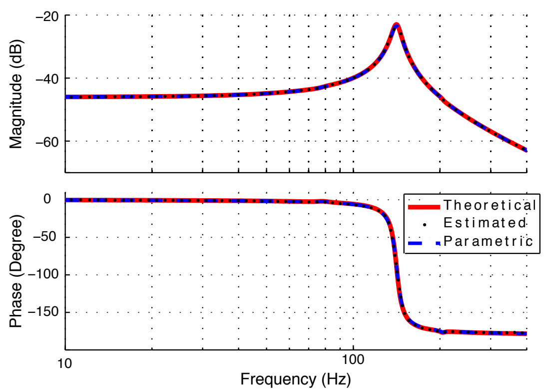

Fig. 4. Estimation results for the fundamental harmonic.

In order to obtain input–output data for system identification, we apply an input signal consisting of nine subsequent $30s$ long chirp signals, each with a linearly increasing frequency in the range $(0, 7]$ Hz over its duration but with a different starting phase evenly distributed across the system's period, $T = 1s$. Each chirp signal has an amplitude of 0.004, chosen to be large enough to perturb system dynamics but small enough to keep the system close to the periodic orbit. A sample chirp signal with zero phase can be generated by

$$u(t) = 0.004 \sin(7\pi t^2/30). \tag{15}$$

The resulting output is then subtracted from the numerically measured limit cycle to obtain error trajectories ξ_1 for vertical position. The input signal and ξ_1 are then used as in [13] to estimate HTFs for our system. Since our theoretical computations showed that responses beyond the third harmonic were very small, we only consider the fundamental harmonic and three harmonics on both sides for our experiments.

Fig. 4 illustrates the estimation performance of our algorithm for the magnitude and phase of the fundamental harmonic. Both graphs show that the application of the identification algorithm in [13] works well even for nonlinear periodic systems with hybrid dynamics.

We also show our identification results for three harmonics in both the negative and positive sides in Fig. 3. Even though magnitudes for the HTFs are small compared to the fundamental, the identification algorithm can provide accurate estimates for these transfer functions except in some narrow regions of G_{-2} and G_2. The identification algorithm could not correctly estimate these two harmonics around $12 - 15$ (rad/s). One possible reason for this discrepancy is the presence of strong responses in all harmonics around the same frequency except G_{-2} and G_2, resulting in the inability of the identification algorithm to distinguish between the contributions from each harmonic absent knowledge of the internal system dynamics. Alternatively, these discrepancies may also be a result of the fact that hybrid transitions are not strictly time periodic (rather, they are state-dependent) which likely has effects on different frequencies and harmonics. We plan on investigating these issues further in the future.

For a comparative analysis, we also present results from a parametric identification in order to show that further corrections on estimation results from a non-parametric method are possible. To this end, we fit the system parameters k and c in (14) by comparing root mean square error between theoretically computed and estimated harmonic transfer functions G_0, G_{-1} and G_1. We truncate the system response after the first harmonic in order to discard erroneous regions in higher harmonics. The resulting estimates were $\hat{k} = 200$ for the spring constant and $\hat{c} = 2.12$ for the damping coefficient, which closely coincide with the parameters used to generate the input–output data. As such, HTFs obtained from parametric identification were found to closely match those obtained from theoretical computations as seen in Fig. 3.

Motivated by these identification results, we plan to extend our work to the identification of the Spring-Loaded Inverted Pendulum (SLIP) model and its extensions, widely used as models of locomotory behaviors in the literature. Our future goal is to apply our system identification methods to our physical monopod robot platform and to compare the identification performances with our previously verified analytical model [10].

V. Conclusion

In this paper, we presented a system identification strategy to estimate input–output transfer functions for a simple hybrid spring mass damper system as a step towards data-driven models for legged locomotion. We first showed that a class of hybrid locomotion models can be approximated with a piecewise constant LTP systems in close proximity to their asymptotically stable limit-cycle. Our analysis and identification framework is based on the concept of harmonic transfer functions (HTFs) [17].

We first observed that the hybrid system dynamics associated with this model exhibits piecewise LTI behavior around its periodic orbit. We then represented this behavior as a purely time periodic system around the limit cycle in order to utilize system identification techniques applicable to Linear Time Periodic systems.

In order to provide a basis for comparison, we computed analytic expressions for HTFs associated with the LTP approximation to our simplified hybrid model. In our theoretical analysis, we considered the system's response up to the 10^{th} harmonic. We observed that there were no meaningful responses on both positive and negative sides after the third harmonic. Therefore, we decided to truncate the system response after the third harmonic during our identification studies.

We then performed systematic simulation studies and identified the HTFs of the same model without knowledge of its internal dynamics. We used an input signal consisting of successive chirp signals, with phases evenly distributed across the system's period, to obtain a full characterization of system dynamics for our frequency range of interest. Our studies showed that LTP system identification techniques can successfully be used to identify the transfer functions of nonlinear periodic models with hybrid system dynamics.

In the future, we plan to extend this work for modeling and identifying locomotor systems with sensory and motor delays. In our previous work, we showed that HTFs allow independent estimation of input and measurement delays for simple locomotion models, such as the one in current paper [24].

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