
2. Log decrement method

(a) Write a computer program (e.g. in Matlab) that implements the logarithmic decrements method. The program should do something such as automatically detect peaks and/or zero crossings, as well as the steady-state value, and then process these in a reasonable way to get the system parameters. Demonstrate your code on “noiseless” step-response data, e.g. by discretely sampling the equation for the step response given in class.

(b) Create a simulation of a second order, under-damped system. You can use ode45, lsim, or build your own integrator (such as simple rectangular integration). Implement your system ID code. Show that it correctly recovers the parameters for the noise-free case.

(c) Set $k = 2$, $\omega_n = 1$, $\zeta = 0.1$ and add 5% measurement noise to the system (that is, an iid Gaussian noise process, whose standard deviation is 5% of the final value).

   i. Write an expression for the autocorrelation function. Assume that it is in continuous time, although of course the numerical method will be for discrete time.

   ii. Run 100 instantiations of your code, and produce a histogram of the parameters. Is your estimator biased? Is your estimator consistent?

   iii. Repeat the previous problem for noise levels from 1 to 20%. If the system is biased, plot the bias as a function of the noise level, from 1% to 20% noise. This may require running lots (and lots) of simulations.

3. Consider a system that is a pure delay, expressed as

$$y(t) = u(t - T)$$  \hspace{1cm} (1)

where $T$ is the delay.

(a) Consider the inputs $u(t) = \cos 2\pi T/3$ and $u(t) = u_s(t)$ (unit step function), and plot $y(t)$.

(b) Find $h(t)$, the impulse response function, so that

$$y(t) = \int_0^t h(t - \tau)u(\tau)d\tau$$  \hspace{1cm} (2)

(c) Take the Laplace transform of both sides of (2) and show that $Y(s) = e^{-sT}U(s)$.

(d) Look up what a Padé approximation is. Discuss why a Padé approximation to the delay might be better than, for example, a simple Taylor expansion approximation such as $e^{-sT} \approx 1 - sT - \frac{1}{2}s^2T^2$. 
