ME 530.676: Locomotion in Mechanical and Biological Systems
Final Project

Instructor: Noah J. Cowan

Due: Thursday May 8 at 11:59:59 pm

The goal of this project is to generate a numerical approximation to a verticle hopper using Discrete-Time Harmonic Transfer Functions. Specifically, you will attempt to reproduce frequency response functions (FRFs) similar to [1], but with a smaller number of sections, and based on a technique that is different than in that paper. In that paper authors produce an HTF estimate based on input–output responses. Here, you'll estimate the dynamics

$$x[k+1] = A[k]x[k] + B[k]u[k]$$
$$y[k] = C[k]x[k]$$

where each of the matrices A, B, C are periodic in period $T = 4$.

The hopper code is a modification of the SLIP code, where hopping is purely in the vertical direction. Start with these parameters:

```
hopper_params.kc=1000;    %compression phase spring constant - N/m
hopper_params.kd=1000;    %decompression phase spring constant - N/m
hopper_params.m=3;        %mass of the model - kg
hopper_params.l_rest=1;   %the rest length of the spring - m
hopper_params.l_td = 1;   %touchdown leg length - m
hopper_params.l_lo = 1;   %liftoff leg length - m
hopper_params.gs=9.81;    %the gravitational accelaration during stance phase m/s2
hopper_params.gf=9.81;    %the gravitational accelaration during flight phase m/s2
hopper_params.Bc=5;       %compression phase damping constant  Ns/m
hopper_params.Bd=5;       %decompression phase damping constant  Ns/m
hopper_params.td_angle=0.0; %Touchdown angle
hopper_params.u0 = [2 2 0 0];
```

What you will do:

1. Find the limit cycle.
2. Use central difference to estimate section maps and the apex return map. Show that the product (in the right order!) of the 4 section maps equals the return map.
3. Use central difference approximation to compute linearized B and C matrices.
4. Find the HTFs from these.
5. Use Bode Plots to show that these agree with Mert's results (I'll provide stub code to help).
6. Turn in a single report and code to me. The report should be 5 or so pages with figures, and should explain your work. It should be typed up and clear.

References

[1] M. M. Ankarali and N. J. Cowan. System identification of rhythmic hybrid dynamical systems via discrete time harmonic transfer functions. In *Submitted to IEEE International Conference on Decision and Control (CDC)*, December 2014.