ME 530.676: Locomotion in Mechanical and Biological Systems
Problem Set 6
Due Friday 18 Apr 2014 in Class
REV 1: UPDATED 14 APR 2014, 10:35PM

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April 14, 2014

1. Consider the following system:

$$\begin{aligned} \dot{x} &= Ax + B(1 + \cos(\omega_0 t))u \\ y &= Cx \end{aligned}$$

where $u, y \in \mathbb{R}$ (single-input, single-output, or SISO) and $x \in \mathbb{R}^2$. Specifically

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -2 \end{bmatrix}$$

   (a) What is the pumping frequency?
   (b) Compute an impulse response function.
   (c) Compute the HTF in terms of transfer functions $\ldots, G_{-2}, G_{-1}, G_0, G_1, G_2, \ldots$.

2. Consider the following system:

$$\begin{aligned} \dot{x} &= Ax + B\beta(t)u \\ y &= \delta(t)Cx \end{aligned}$$

where $u, y \in \mathbb{R}$ (SISO), and A, B, C are constant, and the scalar functions β and δ are both periodic with period T.

   (a) Find a general expression for the impulse response function.
   (b) Given the FS expansion of $\beta(t)$ and δ, with coefficients b_k and d_k, respectively, compute as compact of an expression as possible for the HTF as possible. *Hint: You'll need to look up the time-shift property for FS, as well as the FS coefficients for the product of two periodic functions.*

3. Given a discrete-time, SISO, period-two system of the form

$$\begin{aligned} x[i+1] &= A[i]x[i] + B[i]u[i] \\ y[i] &= C[i]x[i] \end{aligned}$$

with (**UPDATED REV 1**)

$$A[0] = \begin{bmatrix} 0 & 3 \end{bmatrix} \qquad B[0] = \begin{bmatrix} 1 \end{bmatrix} \qquad C[0] = \begin{bmatrix} 1 & -2 \end{bmatrix}$$

$$A[1] = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad B[1] = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad C[1] = \begin{bmatrix} 1 \end{bmatrix}$$

Compute the DTHTF.