

**Transform Table**

	Time	Frequency
<p><b>Laplace Transform (LT)</b></p> <p><math>\omega</math> continuous and aperiodic  <math>t</math> continuous and aperiodic</p>	$x(t) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} X(\omega) \cdot e^{(\gamma-j\omega)t} d\omega$	$X(s) = \int_0^{\infty} x(t) e^{-st} dt$
<p><b>Fourier Transform (FT)</b></p> <p><math>\omega</math> continuous and aperiodic  <math>t</math> continuous and aperiodic</p>	$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$	$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$
<p><b>Fourier Series (FS)</b></p> <p><math>\omega(k)</math> discrete and aperiodic  <math>t</math> discrete and periodic</p>	$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jkt}$	$c_k = \frac{1}{2\pi} \int_0^{2\pi} x(t) e^{-jkt} dt$
<p><b>Discrete Time FT (DTFT)</b></p> <p><math>k</math> (frequency) continuous and periodic  <math>n</math> (time) discrete and aperiodic</p>	$x[n] = \frac{1}{2\pi} \int_0^{2\pi} X(\omega) e^{j\omega n} d\omega$	$X(k) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$
<p><b>Discrete Fourier Transform (DFT)</b>  “Typical” way</p> <p><math>k</math> (frequency) discrete and periodic  <math>n</math> (time) discrete and periodic</p>	$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi nk}{N}}$	$X(k) = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi nk}{N}}$
<p><b>Discrete Fourier Transform (DFT)</b>  “Ljung” way</p> <p><math>\omega</math> (frequency) discrete on the interval <math>(-\pi, \pi)</math>,  and <math>\omega</math> is <math>2\pi</math>-periodic  <math>n</math> (time) discrete <math>\{1, \dots, N\}</math> and periodic</p>	$x[n] = \frac{1}{\sqrt{N}} \sum_{k=1}^N X\left(\frac{2\pi k}{N}\right) e^{j \frac{2\pi nk}{N}}$	$X(\omega) = \frac{1}{\sqrt{N}} \sum_{n=1}^N x[n] e^{-j\omega n}$

**Some DFT Properties (“Standard” – will update)**

<b>Time</b>	<b>Property</b>	<b>Frequency</b>
$x(n - m)$	Time Shift	$X(k)e^{-j\frac{2\pi m}{N}}$
$a.x[n] + b.y[n]$	Linearity	$a.X(k) + b.Y(k)$
$x[-n] = x[N - n]$	Time Reverse	$X(-k) = X(N - k)$
$x^*[n]$	Conjugation	$X^*(-k) = X^*(N - k)$
$x[n - rN]^1 = x[n]$	Periodicity	$X(k - rN) = X(k)$
$x[n] * h[n]$	Convolution	$X(k)H(k)$
$x[n]y[n]$	Multiplication	$\frac{1}{N}X(k) * Y(k)$
$\sum_{n=0}^{N-1}  x[n] ^2$	Parseval	$\frac{1}{N} \sum_{n=0}^{N-1}  X(k) ^2$

**Some DFT Results**

$DFT_N[\delta[n]]$	1
$DFT_N[1]$	$N\delta(k)$
$DFT_N\left(e^{j\frac{2\pi r}{N}}\right)$	$N\delta(k - r)$
$DFT_N[\cos\left(\frac{2\pi r}{N}\right)]$	$\frac{N}{2}\delta(k - r) + \frac{N}{2}\delta(N - k - r)$
$DFT_N[\sin\left(\frac{2\pi r}{N}\right)]$	$\frac{N}{2j}\delta(k - r) + \frac{N}{2j}\delta(N - k - r)$

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<sup>1</sup> For r an integer value