M.E. 530.485 Problem Set 2 (v2)

Noah J. Cowan*
Department of Mechanical Engineering
G.W.C. Whiting School of Engineering
Johns Hopkins University

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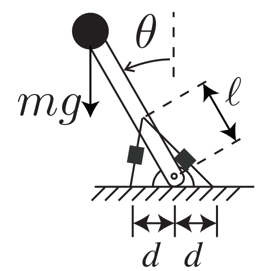
1. Consider an inverted pendulum of length L, with mass m, that is actuated by an agonist/antagonist muscle pair that attach a distance ℓ from the joint / pivot point. One can show



$$\ddot{\theta} - \frac{g}{L}\sin\theta = \frac{1}{mL^2}\tau(t), \tag{1}$$

where $\tau(t)$ is the *net moment* that results from forces applied by the muscles.

   (a) Suppose the left and right muscles produce linear contractile forces F_L and F_R, respectively. Based on the geometry, calculate τ_L and τ_R, the moments due to the left and right muscles, respectively. (Note that τ_R will be negative when F_R is positive, due to the fact that it has a negative moment arm.) Compute the net moment, $\tau = \tau_L + \tau_R$.

   (b) Show that for $\ell \gg d$, we have the following simplification:

$$\tau \approx (d\cos\theta)u(t) \tag{2}$$

   where $u(t) = \Delta F(t) = F_L(t) - F_R(t)$, the difference between the forces applied by the muscles.

   IMPORTANT: For the subsequent problems, use Eq. (2) for the torque unless you want a nightmare of a calculation.

   (c) Combine Eq. (1) with (2) and transform the system into state-space form. You should have a nonlinear equation of the form

$$\dot{x} = f(x, u) \tag{3}$$

   where you must define an appropriate state vector, $x \in \mathbb{R}^2$.

   (d) Make a small-angle approximation to linearize the state-space form. You should have an equation

$$\dot{x} = Ax + Bu \tag{4}$$

   where $A \in \mathbb{R}^{2\times 2}$ and $B \in \mathbb{R}^{2\times 1}$.

   (e) Linearize again, the "hard way" by using the Taylor expansion of $f(x, u)$ and compare with the small-angle apprximation above.

   (f) Compute the transfer function $P(s) = \Theta(s)/U(s)$. Call this the "plant". Find the poles. *Hint, you should have two real poles, $\pm p$, of equal magnitude.* Is the system stable or unstable and why?

(g) Design a PD controller so that the closed-loop "linear" system is stable.

(h) Simulate the closed-loop *linear* dynamics in Matlab using the "shell" code provided for a set of different desired target angles. You should edit and run the "liner_run_student.m". Comment on the results.

(i) Simulate the closed-loop *non-linear* dynamics with the same PD controller and same set of target angles you have tested for the linear case. You should edit and run the "simulation_run.m". Compare the results of the linear and non-linear simulations.